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Research Article

Evaluating University Reputation Based on Integral Linear Programming with Grey Possibility

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As the Chinese economy becomes stronger and stronger, there is increasing interest by some countries to adopt the Chinese approach to grow. The outstanding Chinese growth can be partly attributed to the education quality that is obtainable from the universities in China. The need to evaluate the reputation of Chinese universities is essential. In this paper, a hierarchical model for evaluating universities is proposed. Also, the Integral Linear Programming with Grey Possibility (ILP-GP) method is presented, which does not require the assumption of 0.5 for the grey distinguishing coefficient ($\zeta = 0.5$) in Grey Relational Analysis (GRA). In this paper, a numerical example for evaluating the reputation of five universities based on the sample data from 1,565 students in Shaanxi Province, China, is presented, and the most reputable university in this research is consistent with rankings in the literature.

1. Introduction

The increasing globalization of universities is sometimes credited to the viewpoint of individuals with many changes and difficulties that educational sector has to face. The differences in opinion by politicians, school leaders, and various commentators agree that competition between higher education institutions has intensified over the last few years [1]. Education has a huge role in the development of a country. The progress of education in a country partly symbolizes the general growth of that country. In China, universities are still undergoing profound changes to meet the aspirations and demands of its growth in line with international standard. The 985 and 211 project for educational reform were focused on establishing reputable universities in China [2]. Today more than ever, universities in China, Asia, and rest of the world are competing to attract the best lecturers, students, and substantial number of grants to fund research projects. President Xi Jinping of the People's Republic of China invoked the ancient Silk Road when announcing the "One Belt One Road" initiative in the fall of 2013, and there have been numerous programs in the universities to promote their education advantage, thereby increasing their reputation. Academic Ranking of World Universities (ARWU) developed by Shanghai Jiao Tong University, China, is among the popular international university rankings to keep assessing the reputation of universities in China and other parts of the world.

Reputation is described as the sum of beliefs, ideas, and impressions that a person has about an object, individual, institution, or organization based on past and current events. University Reputation (UR) is an institutional status built as people constructs regarding the university objectives, ethics, working methods, and treatment received by students. In other words, UR refers to a natural, spontaneous character, expectations, and the exchange that people have with the university. Cole and Bruch [3] defined UR as the vision, representation, or impression that people form in their mind in function of the information or data of a university obtained through the interaction with the elements or components of the university. Thus, universities that attract talented human resources or clients are those that maintain and have a good reputation.

Now, rankings are becoming a necessary part of the university's reputation and brand image, helping them attract students, staff, and research investments. Evaluating and ranking of universities can be directly approached as a Multi-Criteria Decision-Making (MCDM) problem. There are uncertainties when assessing the performance of universities based on the various measurement criteria. These uncertainties are addressed using the Grey System Theory (GST) by representing the performances of measurement variable as grey numbers. Decision-making with several restrictions is represented as constraints that are optimized to an objective function. Mathematically, Linear Programming (LP) is a technique that allows the optimization of an objective function through the application of constraints [4]. The main objective of LP is to optimize a linear function, i.e., to maximize or minimize linear functions in several real variables with linear constraints (linear inequalities systems). The LP models contemplate that the decision variables (i.e., the objective function and the constraints) maintain a behaviour of linear type. The objective function and constraints are the component that constitutes the linear functions. Also, uncertainties under various constraints are considered in evaluating UR.

This paper presents two main contributions: firstly, a hierarchical model for evaluating the reputation of universities and secondly, the Integral Linear Programming with Grey Possibility (ILP-GP) method, an optimization method for solving MCDM problems using Grey Relational Analysis (GRA) that do not require the grey distinguishing coefficient (ζ) that is difficult to estimate. These two contributions are applied in evaluating five universities where the universities evaluated are pairwise compared to each other as a maximization function using grey numbers as possibilities based on the GST. This paper is presented as follows: Section 2 gives some related works, Section 3 is the methodology, Section 4 gives the results and analysis, and Section 5 is the conclusion.

2. Related Works

2.1. University Reputation (UR). Universities are social and educational institutions essential to form individuals and the community. Higher education has undergone significant changes from the origin of universities to their recent reform [5]. Reputation refers to the total impression that a person constructs in his/her mind about something or someone [3]. Reputation involves the beliefs, attitudes, stereotypes, ideas, appropriate behaviours, and impressions that a person has of an object, a person, or an organization, while UR is a complex construct based on the perception of the public or personnel of an organization that makes a differentiating and comparative assessment of its characteristics [6]. This status is formed by individuals or by the public as a result of their interpretation of the information or disinformation of a university [7]. UR is the shared knowledge that people have about a university and how it should operate. Also, UR can be seen as perceived external prestige, corporate reputation, and identity [8]. UR has been established as the strategic factor of institutional character to measure the credibility of institutions. Manfredi et al. [9] explained that the need to earn money is no longer enough to survive in the complex and competitive world of the market economy in the twenty-first century, but the value image and the reputation get an essential role in a strategic institutional approach. Saleem et al. [10] studied the moderating role of university culture reputation and price in education sector with a case study of Pakistan by using the service quality and student satisfaction as the criteria from 20 higher educational institutions. They concluded that university culture positively strengthens service quality and student satisfaction, while UR and price negatively strengthened the relationship. Published ranking generates significant pressure on universities around the world to appear in them or in the relevant ministries [11].

According to Proulx [12], the use of indicators for benchmarking purposes always presupposes the choice of comparable universities, i.e., institutions with a common mission, vision, similar strategic objectives, and programs of identical size. The best universities are not the best in all areas. For example, the ranking service by Quacquarelli Symonds (QS) intelligence unit provides a reliable comparison tool for the performance of comparable institutions [13]. ARWU publishes global research universities profiles, a database that can be used as a benchmarking tool, comparing research universities internationally on 40 indicators. The Times Higher Education (THE) [14], QS world university ranking, or multirank defines the trend change that has occurred; the struggle for attraction and loyalty of the best resources is the key that explains the survival of the institution in the medium and long term [15]. However, its validity is often questioned due to the existence of few indicators of internationally comparable quality of higher education [1]. Baty [16] concluded that classifications do not constitute a measure of the absolute value of the university since they do not evaluate whether these universities meet their institutional objectives.

Currently, the typical ranking systems for universities are ARWU, THE, and QS. ARWU takes four criteria for measuring "quality of education": the number of former students (10%) and teachers (20%) who received a Nobel Prize or a Field Medal, the citations of teachers in the 21 thematic categories (20%), universities exit from university research (40%), and universities per capita performance (20%) resulting from the division of the three previous criteria among the number of full-time professors in each study center. The QS gives 40% of the total score to an opinion poll of 15,000 people and 10% of 5,000 surveys of managers and HR managers of companies; 20% comes from student's satisfaction surveys, 20% comes from SciVerse Scopus index scientific publications, and the remaining 10% comes from the number of foreign teachers and students. The "THE" combines surveys with statistical data: surveys represent 60% of the score, 30% comes from the citations of the research, and the remaining 10% is divided among the number of students, foreign teachers, and the income the research receives of each industry. Another ranking system for universities includes University Rankings based on Academic Performance (URAP) [17] and Performance Ranking of Scientific Papers for World Universities (PRSP-WUN) by Taiwanese University Rankings [18]. The URAP and PRSPWUN release the ranking of universities based on academic publications.

Table 1: Summary of the universities ranking system.

No.	Rankings index	Established dates	No. of countries	No. of universities	Methods	Indicator systems
1	ARWU	2003	46	1,200	WSM	Quality of Education, Quality of Faculty, Research Output, Per Capita Performance
2	CWTS Leiden	2007	54	903	Counting Method	Impact Indicators, Collaboration.
3	PRSPWUN	2012	68	800	Weighted Bibliometric	Research Productivity, Impact, and Excellence.
4	QS	1990	Arab Region, Emerging Europe, Central Asia and 5 Brics Countries	916	Peer Review Data	Engineering, Biomedicine, Natural Sciences, Social Sciences and Humanities
5	THE	2009	133	980	Standardize Total Scores	Teaching, Research, Citations, International Outlook, Industry Income.
6	U-MULTI RANK	2011	99	1,500	Classification by Weighted Scores	"Very good" Through to "Weak", In the categories of General, Teaching & Learning, Research, International Orientation, Regional Engagement, Knowledge Transfer.
7	URAP	2010	108	2,500	Bibliometric Total Scores	No. of Articles, Citation, Documents, Article Impact Total, Citation Impact Total, and International Collaboration.

The emergence of rankings imposes challenges for higher education, and research institutions intensify cross-country comparisons. Rankings quickly attracted the attention of universities and politicians as they challenged opinions on reputation and excellence [19]. Verčič et al. [20] explored academic reputation and compared their perceptions and work towards consistent reputation with 25 in-depth interviews with members of key stakeholder's groups. The high competition among universities for attracting students, public resources, social demand for transparency and competence use of funds has resulted in a growing interest in evaluating their performance through rankings. Another study by Moed [21], comparing five world university rankings, provided more insight into the value and limits of world university ranking, by using a comparative analysis; he used ARWU, CWTS Leiden [22], THE, QS, and U-multirank. He showed the overlap in institutional coverage, geographical coverage, the skewness of indicator distributions, and statistics between indicators. Table 1 summarizes the reviewed universities ranking systems.

2.2. Grey System Theory (GST). GST is China's theory developed by Professor Deng Ju Long founded in 1982. GST studies poor information modelling, which provides a new way to solve problems in the case of inadequate information. GST considers all random process as a change in a specific range, which is related to a grey process. GST is not from the perspective of looking for statistical laws, through a large sample of research, but raw data are organized into a regular sequence and then studied. GST deals with behaviour of the system, and chaotic data must heed to a certain law. The GST has evolved to new disciplines that includes the theoretical system based on the grey hazy set, the grey system based on grey correlation space, the grey system based on grey model which is core modelling for system analysis, evaluation, forecasting, decision-making, control, and so on. Grey set, grey algebra system, grey equation, and grey matrix are all based on GST. Although GST has made significant contribution in lot of disciplines, the grey distinguishing coefficient is a problem worthy of further study [23, 24].

Deng [25] introduced GST and gave the grey distinguishing coefficient between the interval of 0 and 1, $\zeta \in (0, 1)$. After over two decades, there is no explicit method accepted for calculating the grey distinguishing coefficient, and the difficulty in determining the grey distinguishing coefficient is self-evident. The problem of determining the grey distinguishing coefficient has been a severe problem, so researchers have to settle for a generalization that the distinguishing coefficient is 0.5, which is the mid-point value within the interval of 0 and 1. Çaydaş and Hasçalık [26] in their work stated that a grey distinguishing coefficient of 0.5 is normally used and referenced Yang [27] which equally stated 0.5 is used for the grey distinguishing coefficient. Ghetiya et al. [28] applied GRA in evaluating friction stir welding process of aluminium alloy and stated if all the evaluation criteria have equal preferences a grey distinguishing coefficient of 0.5 is taken. Most references used 0.5 as the grey distinguishing coefficient, there is no evidence to support the use of 0.5, and the rankings of alternatives are dependent on the grey distinguishing coefficient [28–33]. In this paper, the ILP-GP approach is used in evaluating grey numbers because this approach does not require the use of a grey distinguishing coefficient.

3. Methodology

The hierarchical structure for evaluating UR is deduced from existing related works. The hierarchical structure is a formative construct consisting of first-level and second-level indicators. However, the measured variables of the second-level indicators are reflective constructs. After the criteria weights are assigned by experts, the ILP-GP method is used for ranking the alternatives.

3.1. Evaluation Criteria. Universities are now under pressure to quantify the objective of their performances which has been primarily imposed within the framework of their strategy. For the evaluation of the UR, several measurement aspects are identified, such as the image of the university [34] and the reputation of universities from different countries [8, 35]. In Figure 1, the goal of this evaluation is UR given at the top of the hierarchy consisting of six first-level indicators (C_1 to C_6) and 18 second-level indicators ($C_{1\text{-}1}$ to $C_{6\text{-}3}$). These criteria are primarily deduced from previous quantitative and empirical researches by Plewa et al. [1], Verčič et al. [20], Vidaver-Cohen [36], and others. The first-level criteria are summarized as follows:

(1) *Social contribution* (C_1): this can be seen as the complex feedback of a university, responding to the demands of the environment, where all the actions of the university have an impact in the community. Social contribution can consist of an institutional vision of the ethical contribution and responsibility, which comprises the extent a university will make impact to the society as an organization (for-profit, not-for-profit, or nonprofit) meeting expectations over time (*Citizenship*, $C_{1\text{-}1}$) [20, 37], and the rate at which students from this institution get a job and employees have something positive to say about the university (*Employment*, $C_{1\text{-}2}$). Alumni associations are the consequences of the solidity of the relationship arising from a satisfactory student experience and they are in better position to say the university reputation has positive influence on their degree (*Alumni*, $C_{1\text{-}3}$) [1]. Furthermore, the perceptions of the university alumni to the extent of learning and usefulness of the knowledge acquired serve as key measure to judge the quality and services rendered by the university to its former students [38].

(2) *Environments* (C_2): these are the academic conditions that directly or indirectly affect the development of students and lecturers. A safe, clean, and pleasant environment for students to learn is basic expectation [39]. *Safety* ($C_{2\text{-}1}$) measures the university's ability to protect students from danger [39]. The place where learning occurs is an important part of the student's experiences, which is the social and environmental responsibility of the university (*Campus Location*, $C_{2\text{-}2}$). A reputable university should exceed its local boundaries to gather students from different countries and cultural backgrounds to play a contributing role in its image and helps students to achieve their personal aims [40]. The providing of electronic-learning platform over the Internet highlights the university is internationally renowned (*International Learning*, $C_{2\text{-}3}$) [20].

(3) *Leadership* (C_3): a reputable university should have a clear vision for development, showing competency and good organization [20, 36]: the quality of teaching resources [41], tasks in the form of various assessments within the year of learning (*Course Materials*, $C_{3\text{-}1}$), and the academic staff that present these materials (*Lecturers*, $C_{3\text{-}2}$). At the same time, it is important to identify indicators that reflect the students' experience, that is, their assessment of university life and the fulfilment of their expectations. The student's perspective must be known and recognized in what counts as university activities from admission date till graduating from the university indicated as the perception based on its organizational characteristics (*Administration*, $C_{3\text{-}2}$) [42]. In addition, *administration* focuses on the dimensions of the universities services and pays attention to tangible services and its impact can attract new students [43].

(4) *Funding* (C_4): UR has a financial role on several points. Reputable universities can raise more funds from the governments and incomes of parents and sponsors and have discounted tuition fees in form of scholarships [44]. The *Income Level of Parents/Sponsors* ($C_{4\text{-}1}$) determines the purchasing power of the education provided by the university that is used in running the university, and most parents are the main financial contributor to their children education [10, 45]. There is the primary cost of receiving education and services that give a lot for money in form of good value for tuition (*Tuition*, $C_{4\text{-}2}$) [20, 46], and the ability to attract top talents through the provision of *Scholarships* ($C_{4\text{-}3}$) because high quality academic standard should provide high quality scholarship [47], which includes scholarship of teaching and learning [48].

(5) *Research and development* (C_5): universities in high-income countries seek to take full advantage of

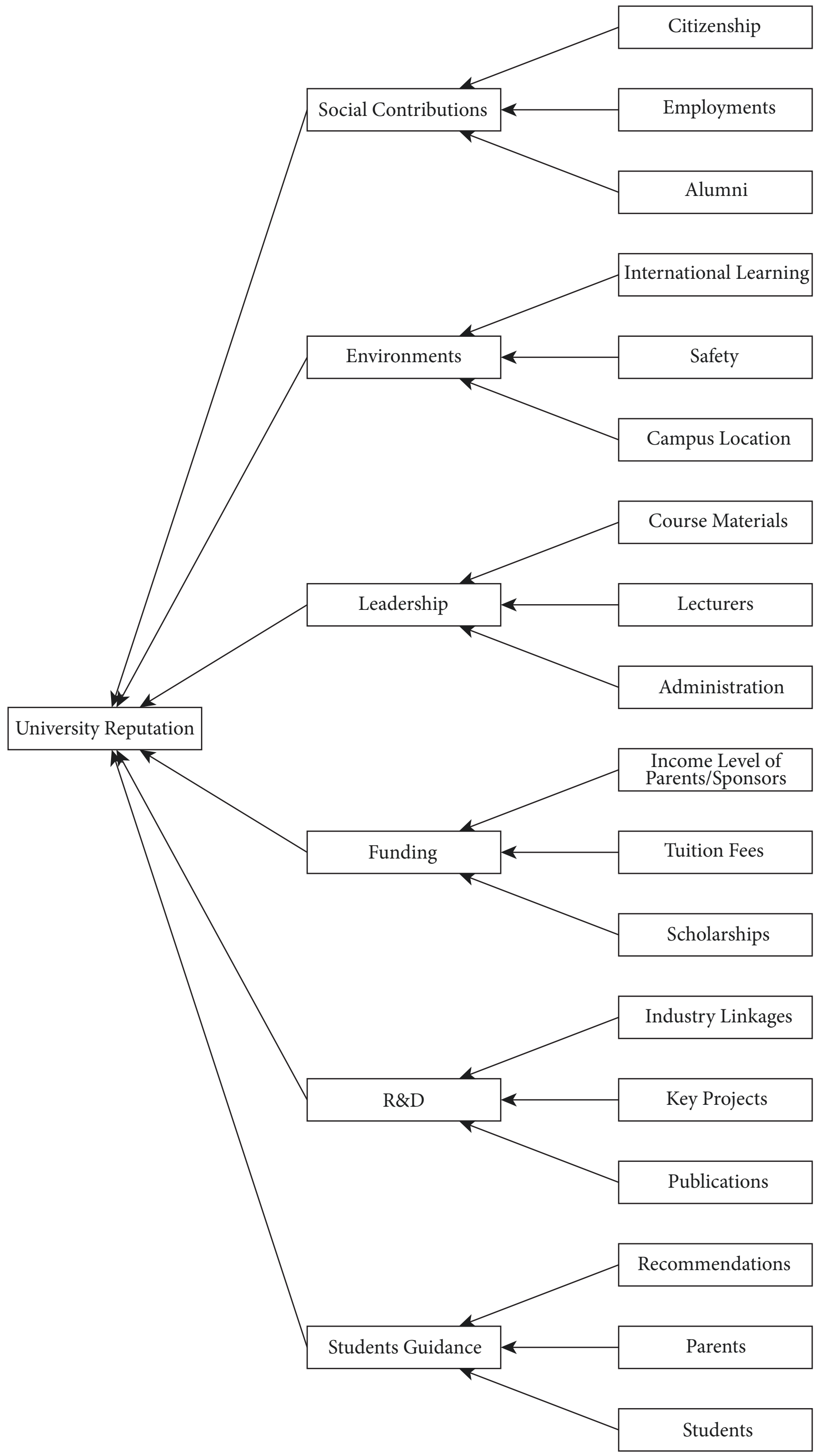


Figure 1: Hierarchical diagram for evaluating UR.

publicly funded research to stimulate knowledge transfer, innovation, entrepreneurship, and economic growth. Industry and university go hand in hand to solve the challenges of innovation for knowledge-based economy that helps in enhancing technological and scientific infrastructure that satisfy the needs of the growing economy (*Industry Linkage*, $C_{5\text{-}1}$) [49, 50]. Also, key research plans from the governments are extended to the universities to provide solutions, where universities are involved in *Key Project* ($C_{5\text{-}2}$) [51, 52]. The research performance of universities can be measured by the number of publications, cited-publications, and international and industry-university copublications [53]. Furthermore, the goals, results, and achievements of these researches are available to the academic community (*Publications*, $C_{5\text{-}3}$) [54].

(6) *Students guidance* (C_6): students are creators of the university life that some have little understanding about when leaving high-school. They receive advice from guidance counsellor, i.e., people who interpret and evaluate a university based on their experience, and the information available about the university. Also, the quality of academic life in a university can predict the university's recommendations, which is a form of the UR (*Recommendations*, $C_{6\text{-}1}$) [55, 56]. Parental control can be a strong influence on children view on UR, and parents use the rankings and reputation of a university as the bases for recommending a university (*Parents*, $C_{6\text{-}2}$) [39, 57]. Other peers are influenced by students who may be their friends indicating their trusts on the university (*Students*, $C_{6\text{-}3}$). Perceived service quality affects students' intention of recommending others to study in their university, and it also affects the students moving to study in any other university [58, 59].

3.2. ILP with Grey Possibilities for Rankings (ILP-GP). The main idea of the ILP-GP is grey interval numbers representing the performances of alternatives under uncertain decision-making environment. The rankings of the alternatives are based on pairwise comparison of the alternatives as grey interval numbers. An approach of comparing two grey numbers is presented as the possibilities of two grey numbers to be inferior or superior. Indeed, for two interval grey numbers to be equal, then the lower bound of both grey numbers must be equal, and the upper bound of both grey numbers must be equal too. The equality of two grey numbers $\otimes A$ and $\otimes B$ is giving as

$$\otimes A = \otimes B, \quad \text{iff } \underline{a} = \underline{b},\ \overline{a} = \overline{b}. \tag{1}$$

Equal grey numbers can be represented in a line graph as shown in Figure 2 where the spotted region is $\otimes A$ and the striped region is $\otimes B$.

Absolute inequalities of two grey numbers are obtainable in a disjointed case. $\otimes A$ is absolutely less than $\otimes B$ when $\overline{a}$ is less than $\underline{b}$. Also, $\otimes A$ is absolutely greater than $\otimes B$ when $\underline{a}$ is greater than $\overline{b}$. That is, $\otimes A < \otimes B$ iff $\overline{a} < \underline{b}$, and $\otimes A > \otimes B$ iff $\underline{a} > \overline{b}$. Figure 3 shows the absolute inequalities in two grey numbers $\otimes A$ and $\otimes B$.

Furthermore, there are other cases of inequalities of two grey numbers as shown in Figure 4, and this raises the question of possibility. In expressing these equalities and inequalities as possibilities, the grey possibility that $\otimes A$ is inferior to $\otimes B$ is denoted as $\otimes A \prec \otimes B$ and the grey possibility that $\otimes A$ is superior to $\otimes B$ is denoted as $\otimes A \succ \otimes B$. The possibility degree is between the intervals of 0 and 1, where $p_{\otimes A \prec \otimes B}$ is the degree of possibility that $\otimes A$ is inferior to $\otimes B$ and $p_{\otimes A \succ \otimes B}$ is the degree of possibility that $\otimes A$ is superior to $\otimes B$. Figure 4 shows the seven conditions of possibilities between two grey numbers $\otimes A$ and $\otimes B$.

Also, note that Li et al. [60] earlier proposed a different kind of grey possibility degree with four conditions for evaluating alternatives, and the weights of the criteria are aggregated using the arithmetic mean. In contrast, seven conditions are present in this paper. Under these conditions, the possibility degree that $\otimes A$ may be inferior to $\otimes B$ is expressed using (2).

$$p_{\otimes A \prec \otimes B} = \begin{cases} 1 & \underline{a} \le \overline{a} \le \underline{b} \le \overline{b} \\ \dfrac{\underline{b} - \underline{a}}{\overline{a} - \underline{a} + 1} + \dfrac{\overline{a} - \underline{b} + 1}{\overline{a} - \underline{a} + 1}\left(0.5\dfrac{\overline{a} - \underline{b} + 1}{b - \underline{b} + 1} + \dfrac{\overline{b} - \overline{a}}{\overline{b} - \underline{b} + 1}\right) & \underline{a} \le \underline{b} \le \overline{a} \le \overline{b} \\ \dfrac{\underline{b} - \underline{a}}{\overline{b} - \underline{a} + 1} + 0.5\dfrac{\overline{b} - \underline{a} + 1}{\overline{b} - \underline{a} + 1} & \underline{a} \le \underline{b} \le \overline{b} \le \overline{a} \\ 0 & \underline{b} \le \overline{b} \le \underline{a} \le \overline{a} \\ 0.5\dfrac{\overline{b} - \underline{a} + 1}{\overline{a} - \underline{a} + 1}\dfrac{\overline{b} - \underline{a} + 1}{\overline{b} - \underline{b} + 1} & \underline{b} \le \underline{a} \le \overline{a} \le \overline{b} \\ 0.5\dfrac{\overline{b} - \underline{a} + 1}{\overline{b} - \underline{b} + 1}\dfrac{\overline{b} - \overline{h}}{\overline{b} - \underline{b} + 1} & \underline{b} \le \underline{a} \le \overline{a} \le \overline{b} \\ 0.5 & \underline{a} = \overline{a} = \underline{b} = \overline{b}. \end{cases} \tag{2}$$

We say $\otimes A$ is inferior to $\otimes B$ when $p_{\otimes A \prec \otimes B} > 0.5$, and $\otimes A$ is superior to $\otimes B$ when $p_{\otimes A \prec \otimes B} < 0.5$. The sum of the inferior and superior possibilities is a unit value; i.e.,

$$p_{\otimes A \prec \otimes B} + p_{\otimes A \succ \otimes B} = 1. \tag{3}$$

For a grey decision matrix X represented as

$$X = \begin{pmatrix} \otimes x_{1,1} & \otimes x_{1,2} & \cdots & \otimes x_{1,n} \\ \otimes x_{2,1} & \otimes x_{2,2} & \cdots & \otimes x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m,1} & \otimes x_{m,2} & \cdots & \otimes x_{m,n} \end{pmatrix}, \tag{4}$$

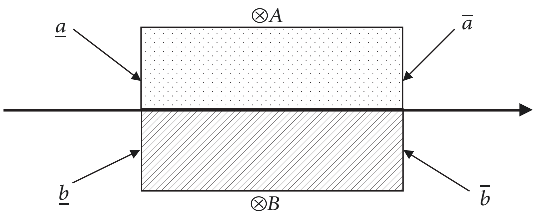


Figure 2: Absolute equalities.

the inferior and superior possibility degrees of matrix X are given as P^{-} and P^{+}, respectively. Also, the sum of the inferior and superior possibility degree matrix and an identity matrix I is a unit matrix,

$$\begin{pmatrix} \otimes p_{11}^{+} & \otimes p_{12}^{+} & \cdots & \otimes p_{1n}^{+} \\ \otimes p_{21}^{+} & \otimes p_{22}^{+} & \cdots & \otimes p_{2n}^{+} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes p_{m1}^{+} & \otimes p_{m2}^{+} & \cdots & \otimes p_{mn}^{+} \end{pmatrix} + \begin{pmatrix} \otimes p_{11}^{-} & \otimes p_{12}^{-} & \cdots & \otimes p_{1n}^{-} \\ \otimes p_{21}^{-} & \otimes p_{22}^{-} & \cdots & \otimes p_{2n}^{-} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes p_{m1}^{-} & \otimes p_{m2}^{-} & \cdots & \otimes p_{mn}^{-} \end{pmatrix} + \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$
$$P^{+} + P^{-} + I = 1. \tag{5}$$

The basic idea in evaluating alternatives based on ILP-GP approach is as follows. First, a grey data matrix based on the evaluation criteria of the alternatives is constructed and normalized. Next, the alternatives based on their criteria are pairwise compared and the grey inferior possibilities are calculated between the pair of alternatives. Then, an ILP objective function is used to optimize the selection of alternatives based on their criteria weights such that if one alternative has more preference, the others are less preferred. Lastly, the alternatives are sorted and ranked. The steps for ILP-GP are as follows.

Step 1. Construct the grey decision matrix: for a decision matrix,

$$\Phi = \begin{pmatrix} \otimes\phi_{1,1} & \otimes\phi_{1,2} & \cdots & \otimes\phi_{1,n} \\ \otimes\phi_{2,1} & \otimes\phi_{2,2} & \cdots & \otimes\phi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi_{m,1} & \otimes\phi_{m,2} & \cdots & \otimes\phi_{m,n} \end{pmatrix}, \tag{6}$$

where $\otimes\phi_{ij} = [\underline{\phi}_{ij}, \overline{\phi}_{ij}]$ is the grey number of the jth criterion of ith alternative. The ith alternative expressed in a vector form is given as $\Phi_i = (\otimes\phi_{i,1}, \otimes\phi_{i,2}, \ldots, \otimes\phi_{i,n})$.

Step 2. Normalize the grey decision matrix. The decision matrix Φ is normalized. For the benefits preferences, i.e., the higher the value then the better the value, they are normalized as follows:

$$\otimes\phi_{ij}^{*} = \left[\frac{\underline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}}{\max_{1\le i\le m}\overline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}}, \frac{\overline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}}{\max_{1\le i\le m}\overline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}} \right]. \tag{7}$$

For the cost preferences, i.e., the smaller the value then the better the value, they are normalized as follows:

$$\otimes\phi_{ij}^{*} = \left[\frac{\max_{1\le i\le m}\overline{\phi}_{ij} - \overline{\phi}_{ij}}{\max_{1\le i\le m}\overline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}}, \frac{\max_{1\le i\le m}\overline{\phi}_{ij} - \underline{\phi}_{ij}}{\max_{1\le i\le m}\overline{\phi}_{ij} - \min_{1\le i\le m}\underline{\phi}_{ij}} \right]. \tag{8}$$

A normalized decision matrix is constructed,

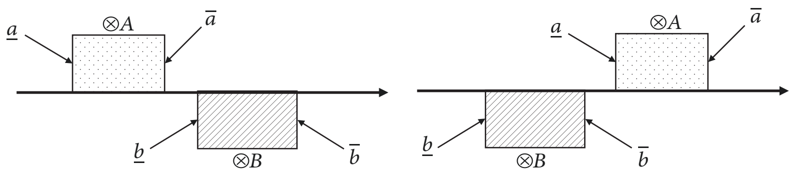
$$\Phi^{*} = \begin{pmatrix} \otimes\phi^{*}{}_{11} & \otimes\phi^{*}{}_{12} & \cdots & \otimes\phi^{*}{}_{1n} \\ \otimes\phi^{*}{}_{21} & \otimes\phi^{*}{}_{22} & \cdots & \otimes\phi^{*}{}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi^{*}{}_{m1} & \otimes\phi^{*}{}_{m2} & \cdots & \otimes\phi^{*}{}_{mn} \end{pmatrix}. \tag{9}$$

In a vector form, we define

$$\Phi_i^{*} = \left(\otimes\phi_{i,1}^{*} \otimes \phi_{i,2}^{*}, \ldots, \otimes\phi_{i,n}^{*}\right). \tag{10}$$

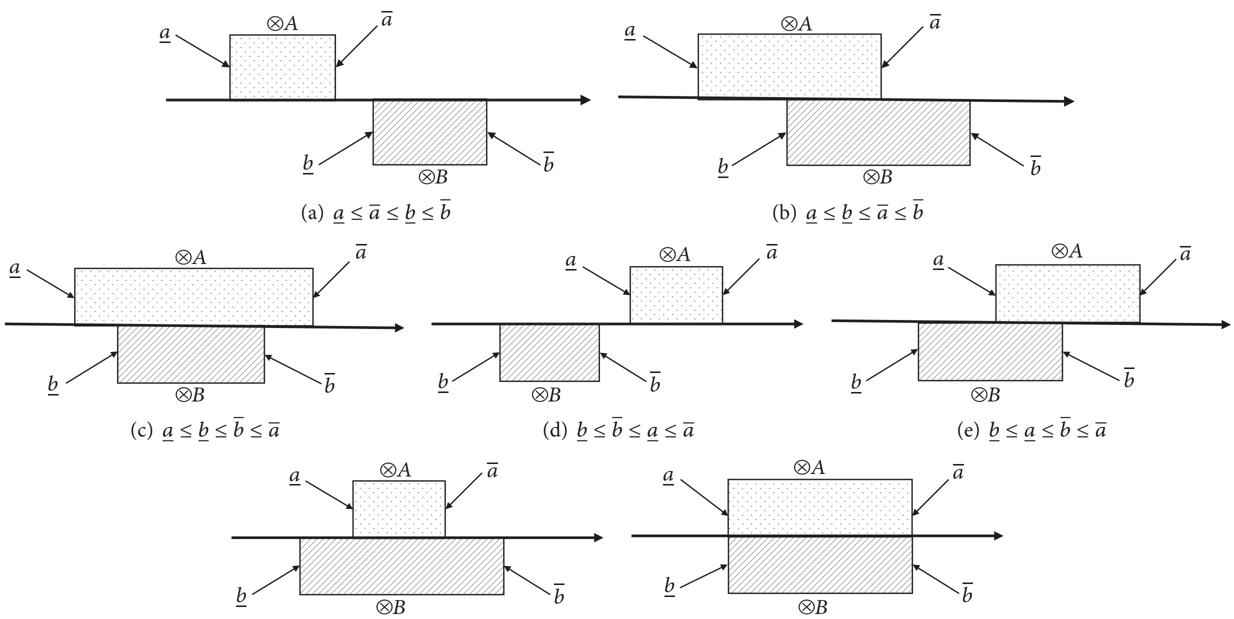
Step 3. Determine the weights of the criteria. Any weighing method in the literature can be used [61].

Step 4. Obtain the grey possibilities and solve the LP problem. Now, the inferior possibilities of the normalized decision matrix Φ^{*} for criterion k and a pair of alternatives i and j are denoted as Q_{ijk}. For two alternatives, $\otimes\alpha = [\underline{\alpha}, \overline{\alpha}]$ and $\otimes\beta = [\underline{\beta}, \overline{\beta}]$, respectively. The possibility alternative that $\otimes\alpha$ is inferior to $\otimes\beta$ is given as



(a) $\otimes A$ is less than $\otimes B$ (b) $\otimes A$ is greater than $\otimes B$

Figure 3: Absolute inequalities.



(a) $\underline{a} \le \overline{a} \le \underline{b} \le \overline{b}$ (b) $\underline{a} \le \underline{b} \le \overline{a} \le \overline{b}$

(c) $\underline{a} \le \underline{b} \le \overline{b} \le \overline{a}$ (d) $\underline{b} \le \overline{b} \le \underline{a} \le \overline{a}$ (e) $\underline{b} \le \underline{a} \le \overline{b} \le \overline{a}$

(f) $\underline{b} \le \underline{a} \le \overline{a} \le \overline{b}$ (g) $\underline{a} = \overline{a} = \underline{b} = \overline{b}$

Figure 4: Grey possibilities.

$$Q_{\otimes\alpha<\otimes\beta} = \begin{cases} 1 & \underline{\alpha} \le \overline{\alpha} \le \underline{\beta} \le \overline{\beta} \\ \dfrac{\underline{\beta} - \underline{\alpha}}{\overline{\alpha} - \underline{\alpha} + 1} + \dfrac{\overline{\alpha} - \underline{\beta} + 1}{\overline{\alpha} - \underline{\alpha} + 1}\left(0.5\dfrac{\overline{\alpha} - \underline{\beta} + 1}{\overline{\beta} - \underline{\beta} + 1} + \dfrac{\overline{\beta} - \overline{\alpha}}{\overline{\beta} - \underline{\beta} + 1}\right) & \underline{\alpha} \le \underline{\beta} \le \overline{\alpha} \le \overline{\beta} \\ \dfrac{\underline{\beta} - \underline{\alpha}}{\overline{\beta} - \underline{\alpha} + 1} + 0.5\dfrac{\overline{\beta} - \underline{\alpha} + 1}{\overline{\beta} - \underline{\alpha} + 1} & \underline{\alpha} \le \underline{\beta} \le \overline{\beta} \le \overline{\alpha} \\ 0 & \underline{\beta} \le \overline{\beta} \le \underline{\alpha} \le \overline{\alpha} \\ 0.5\dfrac{\overline{\beta} - \underline{\alpha} + 1}{\overline{\alpha} - \underline{\alpha} + 1}\dfrac{\overline{\beta} - \underline{\alpha} + 1}{\overline{\beta} - \underline{\beta} + 1} & \underline{\beta} \le \underline{a} \le \overline{a} \le \overline{\beta} \\ 0.5\dfrac{\overline{\beta} - \underline{\alpha} + 1}{\overline{\beta} - \underline{\beta} + 1}\dfrac{\overline{\beta} - \overline{\alpha}}{\overline{\beta} - \underline{\beta} + 1} & \underline{\beta} \le \underline{a} \le \overline{a} \le \overline{\beta} \\ 0.5 & \underline{a} = \overline{a} = \underline{\beta} = \overline{\beta}. \end{cases} \quad (11)$$

The ILP-GP method is formulating an optimization problem using logical constraints that are represented as 0 and 1 variables. Selecting the best alternative is a maximization ILP objective function. The ILP problem has integer constraints $\mu_{ij} = 0$ or 1 which are binary constraints. The decision variables for the pairwise comparison of the alternative can be defined as $\mu_{ij} = 1$ if alternative i is more important than j, and $\mu_{ij} = 0$ otherwise, where $i = 1, 2, 3, \ldots, n$. This means that in comparing two alternatives, one alternative is more important than the other, and it indicates that two alternatives cannot be equally important. In other words, two alternatives cannot coexist as the best alternative, and it is represented as a constraint $\mu_{ij} + \mu_{ji} = 1$; i.e., one alternative must be the best. Also, only two alternatives can be compared at a time, resulting in a pairwise comparison of all alternatives. Moreover, to determine the rankings of the alternatives as a sequence, three alternatives are compared with some conditions. For instance, consider alternatives i, j, and l. After a pairwise comparison of the alternatives i, j, and l, if alternative i is more important than alternative j and alternative j is more important than alternative l, then alternative i must be more important than alternative l, and alternative i is the best alternative. These constraints can be represented as $\mu_{ij} + \mu_{jl} = 2$ and $\mu_{il} = 1$. If these two constraints are satisfied, then the sequence for ranking the criteria is $i > j > l$. Then combining the two equations, a constraint $\mu_{ij} + \mu_{jl} \leq \mu_{il} + 1$ is added. To obtain the sequence for ranking all the alternatives, 3 combination n criteria constraints must be added; i.e., $C_n^3 = n!/3!(n-3)!$

Thus, the best alternative is the alternative with the highest performances based on all the criteria, and this is expressed as a maximization objective function since we need the best alternative. The objective function to obtain the best alternative is represented as the highest value for the summation of the weights of the criteria and the possibilities that the alternatives are more superior, as well as the binary variables of the alternatives that are more preferred than their corresponding pairs. The LP model is defined as follows.

Objective function is

$$\max \sum_{k=1}^{n} w_k Q_{jik} \mu_{ij}, \quad \text{for } i, j \in A \tag{12}$$

subject to

$$\mu_{ij} + \mu_{ji} = 1 \quad \text{for } i, j \in A;\ i \neq j \tag{13}$$

$$\mu_{ij} + \mu_{jl} - 1 \leq \mu_{il} \quad \text{for } i, j, l \in A;\ i \neq j \neq l \tag{14}$$

$$\mu_{ij} \in \{0, 1\} \quad \text{for } i, j \in A;\ i \neq j, \tag{15}$$

where n is the number of criteria, A is the set of alternatives, μ_{ij} is 1 if one prefers A_i to A_j, 0 otherwise, and w_k is the weight assigned to criterion k. The objective function (12) is a maximization function that is the linear aggregation of the criteria weights (w_k) and the possibilities of the alternatives are inferior to each other denoted as Q_{jik}. This objective function imposes that the greater Q_{jik} (i.e., the greater possibility of alternative j being less important than alternative i), the greater possibility that $\mu_{ij} = 1$, which means alternative i is more important than alternative j. The summation of the possibilities for all alternatives that are inferior is zero because if an alternative is less preferred, then $\mu_{ij} = 0$. In comparing two alternatives, either one of the alternatives must be preferred as indicated by constraint (13). Constraint (14) is the sequencing constraint. Constraint (15) indicates that μ_{ij} are binary variables. The method described in this section is the Integer Linear Programming with Grey Possibilities (ILP-GP) approach for rankings [62].

4. Numerical Example

4.1. Sample Data and Screening. The ILP-GP method is applied to evaluate five universities (alternatives): AAA University (A_1), BBB University (A_2), CCC University (A_3), DDD University (A_4), and EEE University (A_5). The names of these universities are kept anonymous for two reasons. Firstly, the criteria weight may be considered subjective. Secondly, this paper is neither to promote the top ranking university nor to tarnish the reputation of the lowest ranked university.

To begin, a questionnaire is developed by adopting some measurement variables from previous works [1, 20, 36]; next the questionnaire was translated to Chinese and piloted tested using a scale of five *(Totally Disagree, Disagree, Indifferent, Agree, and Totally Agree)*. After several revisions of the questionnaire, it is used as our survey instrument. 320 questionnaires were distributed to each of the five universities. 1,592 of 1,600 questionnaires were recovered. 13 questionnaires were unanswered, with 14 questionnaire having unattended responses. Thus 1,565 samples were used for data analysis; i.e., 97.8% of the distributed questionnaire is used for data analysis. The second-level criteria are measured as a reflective construct; i.e., the second-level indicator causes the results of the questionnaire. Figure 5 shows how the second-level indicator *Citizenship* ($C_{1\text{-}1}$) is measured as a reflective construct, and other second-level indicators are omitted. The formative construct in Figure 5 is zoomed in on Figure 1.

Since reflective variables are highly correlated variables, which means not using some of the measured variables will not affect the result, the average of the measured variables for each alternative is calculated, then the minimum and maximum of the average measured variable of the construct are used to represent the lower and upper band of the grey number; i.e.,

$$\otimes\phi = \left[\min_{1\leq\delta\leq\eta} \overline{C}_{\alpha-\beta-\delta}, \max_{1\leq\delta\leq\eta} \overline{C}_{\alpha-\beta-\delta}\right], \tag{16}$$

where α and β are the first- and second-level reference of the criteria C and $\overline{C}$ is the mean of C with the last term η for the measured variable δ.

For instance, for *social contribution* of *Citizenship* indicator ($C_{1\text{-}1}$), the grey value is given in Table 2. Other second-level indicators are similarly measured as shown in Table 3, and the computation is omitted. Table 3 gives the results of the computed grey data of the second-level indicators using (16).

4.2. Weights of Indicators. For this research, the average weights assigned by four *Decision-Makers (DMs)* who are academia from four different universities are used in the evaluation process. These weights are subjective to the perspective of the DMs. Any of the various weighting methods in the literature can be used with the ILP-GP [63, 64].

In this example, direct rating method that is normalized and scaled is used to estimate the weights [65]. First, the DMs are asked to assign percentage scores to all criteria. Next, the

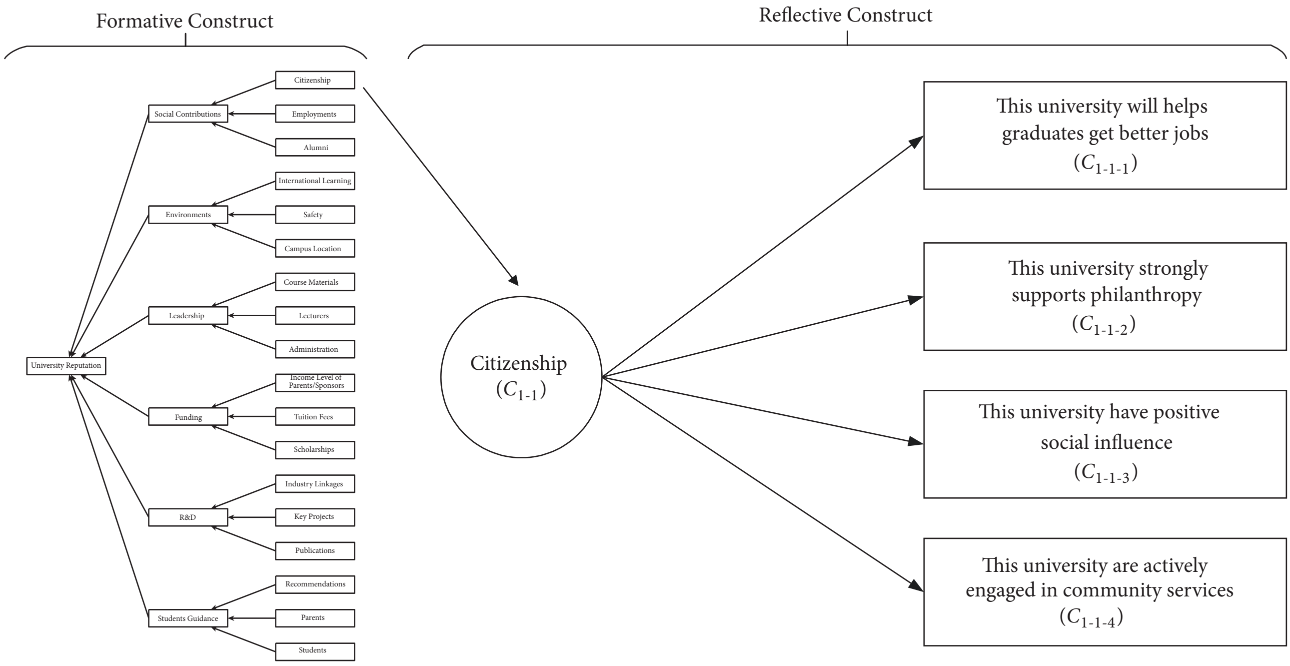


FIGURE 5: Measurement constructs.

TABLE 2: Transformation of sample data to grey numbers.

Measure variable/universities	$C_{1\text{-}1\text{-}1}$	$C_{1\text{-}1\text{-}2}$	$C_{1\text{-}1\text{-}3}$	$C_{1\text{-}1\text{-}4}$	$\otimes\phi_{i1}$
A_1	4.3612	3.9910	4.1104	3.9761	[3.9761, 4.3612]
A_2	4.1378	3.8910	4.4391	4.2436	[3.891, 4.4391]
A_3	4.2000	4.0164	4.4787	4.4000	[4.0164, 4.4787]
A_4	4.1176	3.7451	4.3170	4.0163	[3.7451, 4.317]
A_5	4.2500	3.9333	4.4500	4.2733	[3.9333, 4.45]

TABLE 3: Grey data for evaluating UR.

Φ_{ij}	Φ_{1j} (A_1)	Φ_{2j} (A_2)	Φ_{3j} (A_3)	Φ_{4j} (A_4)	Φ_{5j} (A_5)
Φ_{i1}	[3.9761, 4.3612]	[3.891, 4.4391]	[4.0164, 4.4787]	[3.7451, 4.317]	[3.9333, 4.45]
Φ_{i2}	[3.8583, 4.1164]	[3.3141, 4.1058]	[3.3443, 3.9607]	[3.4052, 3.9281]	[3.6133, 4.2]
Φ_{i3}	[3.9015, 4.2478]	[4.141, 4.266]	[4.1246, 4.282]	[4.0163, 4.2876]	[4.29, 4.4867]
Φ_{i4}	[3.6507, 4.0716]	[3.4263, 4.4263]	[3.3919, 4.4721]	[3.0523, 4.4379]	[4.0233, 4.5667]
Φ_{i5}	[3.9612, 4.1403]	[4.0353, 4.234]	[4.2787, 4.3967]	[4.0817, 4.1667]	[4.1533, 4.3]
Φ_{i6}	[3.7224, 3.9612]	[2.9006, 3.8397]	[3.4984, 3.9082]	[3.268, 4.0654]	[3.43, 4.1333]
Φ_{i7}	[3.7612, 4.0119]	[3.5149, 3.9068]	[3.6689, 3.8787]	[3.3954, 3.6765]	[3.78, 4.0767]
Φ_{i8}	[3.8537, 4.1194]	[4.0128, 4.1571]	[3.9443, 4.0656]	[3.6405, 4.1176]	[4.02, 4.3733]
Φ_{i9}	[3.7851, 4.4119]	[3.6154, 4.1699]	[3.6623, 4.0754]	[3.1961, 3.8725]	[3.5833, 4.2267]
Φ_{i10}	[3.8776, 4.2448]	[2.8942, 3.8237]	[3.1672, 4]	[2.6667, 3.9902]	[3.88, 4.3133]
Φ_{i11}	[3.7522, 4.0478]	[2.7244, 4.1474]	[3.0393, 4.0787]	[3.0425, 4.134]	[3.0333, 4.22]
Φ_{i12}	[3.9552, 4.2269]	[4.0128, 4.3343]	[4.1481, 4.2984]	[3.9739, 4.2843]	[4.2233, 4.3567]
Φ_{i13}	[3.8448, 4.0687]	[3.5385, 3.8846]	[3.7443, 3.8492]	[3.6471, 3.7353]	[3.4833, 4.0633]
Φ_{i14}	[3.8537, 4.3134]	[3.5545, 4.2286]	[3.8098, 4.1738]	[3.6993, 4.4641]	[3.9467, 4.48]
Φ_{i15}	[3.9045, 4.1821]	[3.9167, 4.2083]	[3.6951, 4.0465]	[3.8235, 4.1111]	[4.1733, 4.3167]
Φ_{i16}	[3.7701, 3.8866]	[2.9551, 3.7051]	[3.0623, 3.6098]	[2.8987, 3.781]	[3.2867, 3.9933]
Φ_{i17}	[3.4836, 3.9403]	[2.4103, 3.5417]	[2.6131, 3.5148]	[2.4183, 3.7157]	[2.5133, 3.9433]
Φ_{i18}	[3.7881, 4.0746]	[3.5032, 4.2244]	[3.3508, 4.066]	[3.3072, 4.1176]	[3.77, 4.2367]

Table 4: Direct ratings by the DMs.

Criteria	DM_1	DM_2	DM_3	DM_4	Index (j)
$C_{1\text{-}1}$	90	85	88	90	1
$C_{1\text{-}2}$	98	90	98	95	2
$C_{1\text{-}3}$	95	90	80	90	3
$C_{2\text{-}1}$	85	85	90	95	4
$C_{2\text{-}2}$	100	97	95	97	5
$C_{2\text{-}3}$	90	90	92	98	6
$C_{3\text{-}1}$	95	88	100	95	7
$C_{3\text{-}2}$	100	86	95	96	8
$C_{3\text{-}3}$	98	87	90	99	9
$C_{4\text{-}1}$	80	89	90	95	10
$C_{4\text{-}2}$	80	90	88	90	11
$C_{4\text{-}3}$	80	90	80	92	12
$C_{5\text{-}1}$	80	96	95	93	13
$C_{5\text{-}2}$	85	95	98	95	14
$C_{5\text{-}3}$	85	90	95	94	15
$C_{6\text{-}1}$	80	90	88	90	16
$C_{6\text{-}2}$	85	88	80	85	17
$C_{6\text{-}3}$	80	85	95	95	18
C_1	95	90	80	90	19
C_2	98	90	95	95	20
C_3	100	88	85	95	21
C_4	80	88	90	93	22
C_5	85	97	90	94	23
C_6	75	89	80	90	24
$\min_{1\le j\le 24} x_j$	75	85	80	85	-
$\max_{1\le j\le 24} x_j$	100	97	100	99	-

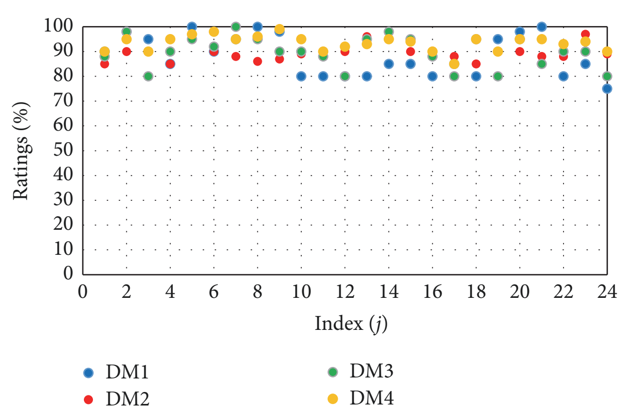


Figure 6: Scatter plot of direct ratings.

scores given by each DM are normalized. Then the average normalized scores are scaled to sum up to a unit value, which is 100%. The steps are given as follows.

Step 1 (obtain the percentage direct ratings for every criterion). The DMs rate the criteria 0–100. The scores are given in Table 4 and Figure 6 is a scatter plot of the direct ratings.

Step 2 (normalize the percentage ratings). The direct ratings assigned are normalized because the scores assigned by the DMs are skewed between 75 and 100%. The formula below is used for normalizing

$$x_j^* = \frac{x_j - \min_{1\le j\le t} x_j}{\max_{1\le j\le t} x_j - \min_{1\le j\le t} x_j}, \tag{17}$$

Table 5: Normalized direct ratings.

Criteria	DM_1	DM_2	DM_3	DM_4	Average ($\overline{x}_j^*$)
$C_{1\text{-}1}$	0.6000	0.0000	0.4000	0.3571	0.3393
$C_{1\text{-}2}$	0.9200	0.4167	0.9000	0.7143	0.7377
⋮	⋮	⋮	⋮	⋮	⋮
C_5	0.4000	1.0000	0.5000	0.6429	0.6357
C_6	0.0000	0.3333	0.0000	0.3571	0.1726

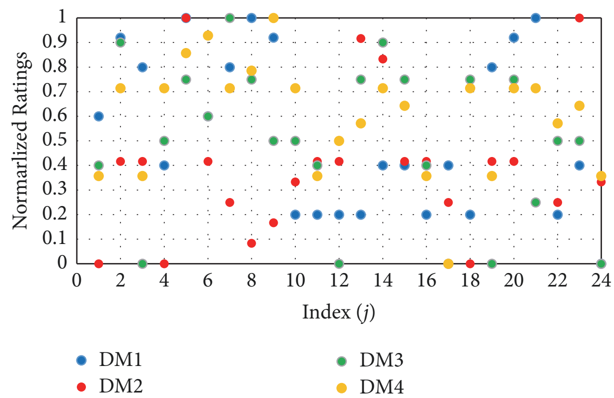


Figure 7: Scatter plot of the normalized direct ratings.

where x is the ratings given by the DMs for criteria j for which the last term is t

$$x_j^* = \frac{x_j - \min_{1\le j\le 24} x_j}{\max_{1\le j\le 24} x_j - \min_{1\le j\le 24} x_j}. \tag{18}$$

The average normalized direct ratings are given as

$$\overline{x}_j^* = \frac{1}{t}\sum_{j=1}^{t} x_j^*; \tag{19}$$

i.e.,

$$\overline{x}_j^* = \frac{1}{24}\sum_{j=1}^{24} x_j^*. \tag{20}$$

The normalized direct ratings are given in Table 5 and Figure 7 shows its scatter plot.

Step 3 (scale the average normalized average ratings). After scaling, the local weights and effective weights are obtained.

$$x_j' = \frac{x_j}{\sum_{j=1}^{n} x_j}, \tag{21}$$

$$\sum_{\alpha=1}^{\eta} x_\alpha = 1, \tag{22}$$

$$\sum_{\beta=1}^{\eta} x_{\alpha\text{-}\beta} = 1. \tag{23}$$

Step 4 (calculate the effective weights of the criteria). The effective weights distribute the effects of each second-level criterion on the overall goal (UR). The product of the first-level criteria weights (W_α) and the local second-level criteria weights ($W_{\alpha\text{-}\beta}$) is the effective second-level weights (W').

$$W' = W_\alpha \times W_{\alpha\text{-}\beta}. \tag{24}$$

The effective weights are given in Table 6.

4.3. *Application of ILP-GP for Evaluation.*

The ILP-GP approach of solving MCDM problem based on grey numbers and the pairwise comparisons of the grey numbers as their possibilities is implemented in C++. The ILP-GP problem is solved using IBM ILOG CPLEX Optimization Studio Version: 12.6.2.0 Build id: 0

The steps are as follows.

Step 1 (construct the grey decision matrix). The values of the second-level criteria are obtained from Table 3, where $\otimes\phi_{i,j}$ is the element of the decision matrix of second-level indicators j for university i obtained from the surveys when $1 \le i \le 5$, and $1 \le j \le 18$.

$$\Phi = \begin{pmatrix} [3.9761, 4.3612] & [3.891, 4.4391] & \cdots & [3.9333, 4.45] \\ [3.8583, 4.1164] & [3.3141, 4.1058] & \cdots & [3.6133, 4.2] \\ \vdots & \vdots & \ddots & \vdots \\ [3.7881, 4.0746] & [3.5032, 4.2244] & \cdots & [3.77, 4.2367] \end{pmatrix}. \tag{25}$$

The first element of the grey decision matrix is $\Phi = \otimes\phi_{1,1}$ and it corresponds to the grey value of the *Foundation* as *Social Contributions* in AAA University (A_1) with a lower bound of 3.9761 units and an upper bound of 4.3612 units. All the elements of the matrix Φ have similar corresponding lower

Table 6: Effective criteria weights for UR evaluation.

First-level indicators, C_α	Average normalized ratings, $\overline{x}_j^*$	First-level weights, W_α (%)	Second-level indicators, $C_{\alpha\text{-}\beta}$	Average normalized ratings, $\overline{x}_j^*$	Local second-level weights, $W_{\alpha\text{-}\beta}$	Effective second-level weights, W' (%)
C_1	0.3935	13.8737	$C_{1\text{-}1}$	0.3393	0.2307	3.2011
			$C_{1\text{-}2}$	0.7377	0.5017	6.9605
			$C_{1\text{-}3}$	0.3935	0.2676	3.7122
C_2	0.7002	24.6915	$C_{2\text{-}1}$	0.4036	0.2078	5.1321
			$C_{2\text{-}2}$	0.9018	0.4644	11.4677
			$C_{2\text{-}3}$	0.6363	0.3277	8.0917
C_3	0.5536	19.5198	$C_{3\text{-}1}$	0.6911	0.3468	6.7702
			$C_{3\text{-}2}$	0.6548	0.3286	6.4145
			$C_{3\text{-}3}$	0.6467	0.3246	6.3351
C_4	0.3804	13.4120	$C_{4\text{-}1}$	0.4369	0.4124	5.5306
			$C_{4\text{-}2}$	0.3435	0.3242	4.3476
			$C_{4\text{-}3}$	0.2792	0.2635	3.5338
C_5	0.6357	22.4163	$C_{5\text{-}1}$	0.6095	0.3253	7.2917
			$C_{5\text{-}2}$	0.7119	0.3799	8.5165
			$C_{5\text{-}3}$	0.5524	0.2948	6.6081
C_6	0.1726	6.0868	$C_{6\text{-}1}$	0.3435	0.3725	2.2673
			$C_{6\text{-}2}$	0.1625	0.1762	1.0728
			$C_{6\text{-}3}$	0.4161	0.4513	2.7467
Total	2.8356	100	-	9.2600	6	100

and upper bounds to the second-level indicators for every university.

Step 2 (normalize the grey decision matrix). A normalized grey decision matrix is constructed using (9),

$$\Phi^* = \begin{pmatrix} [0.6017, 0.7808] & [0.5747, 0.6739] & \cdots & [0.5373, 0.6627] \\ [0.6763, 0.8606] & [0.5115, 0.7473] & \cdots & [0.5609, 0.7853] \\ \vdots & \vdots & \ddots & \vdots \\ [0.6822, 0.8522] & [0.5647, 0.7753] & \cdots & [0.6308, 0.7761] \end{pmatrix}. \tag{26}$$

Based on (10), a vector form of matrix Φ^* is obtained.

For example, $\Phi_1^* = ([0.6017, 0.7808], [0.5747, 0.6739], [0.5988, 0.7134], [0.4923, 0.6622], [0.5973, 0.6644], [0.5313, 0.6241], [0.5306, 0.6296], [0.5684, 0.6672], [0.5318, 0.7893], [0.5687, 0.7177], [0.5328, 0.6368], [0.5831, 0.7182], [0.5604, 0.6602], [0.56, 0.7475], [0.5659, 0.6903], [0.5373, 0.5876], [0.44, 0.6067], [0.5373, 0.6627])$. Other data are omitted here.

Step 3 (determine the weights of the criteria). The weights are the average weights assigned by the experts presented in Section 4.2. For all the second-level indicators weights are used. See Table 6.

$$W' = (3.2011, 6.9605, 3.7122, 5.1321, 11.4677, 8.0917, 6.7702, 6.4145, 6.3351, 5.5306, 4.3476, 3.5338, 7.2917, 8.5165, 6.6081, 2.2673, 1.0728, 2.7467)^{T}. \tag{27}$$

Step 4 (obtain the grey possibilities and solve the LP problem). Based on the procedure in Section 3.2, the five universities are ranked using the second-level indicators and weights, W'.

Objective function is

$$\max \sum_{k=1}^{18} w_k Q_{jik} \mu_{ij}, \quad \text{for } i, j \in A,\ 1 \le i \le 5,\ 1 \le j \le 5 \tag{28}$$

subject to

$$\begin{aligned} &\mu_{ij} + \mu_{ji} = 1 \quad \text{for } i, j \in A;\ i \ne j, \\ &\mu_{ij} + \mu_{jl} - 1 \le \mu_{il} \quad \text{for } i, j, l \in A;\ i \ne j \ne l, \\ &\mu_{ij} \in \{0, 1\} \quad \text{for } i, j \in A;\ i \ne j. \end{aligned} \tag{29}$$

The objective function indicates there are 18 second-level indicators and 5 alternatives to be assessed. After solving by CPLEX, the generated LP function has 20 binary variables and 80 constraints as summarized below.

Objective function is

$$\begin{aligned} \max\ & 0.6668\mu_{1,2} + 0.3332\mu_{1,3} + 0.6469\mu_{1,4} \\ & + 0.3531\mu_{1,5} + 0.5737\mu_{2,1} + 0.4261\mu_{2,3} \\ & + 0.7457\mu_{2,4} + 0.2543\mu_{2,5} + 0.5612\mu_{3,1} \\ & + 0.4388\mu_{3,2} + 0.4846\mu_{3,4} + 0.5154\mu_{3,5} \\ & + 0.5415\mu_{4,1} + 0.4585\mu_{4,2} + 0.3904\mu_{4,3} \\ & + 0.6096\mu_{4,5} + 0.5588\mu_{5,1} + 0.4411\mu_{5,2} \\ & + 0.6662\mu_{5,3} + 0.3338\mu_{5,4} \end{aligned} \tag{30}$$

subject to

$$\begin{aligned} &\text{Constraint 1: } \mu_{1,2} + \mu_{1,3} = 1 \\ &\text{Constraint 2: } \mu_{1,4} + \mu_{1,5} = 1 \\ &\text{Constraint 3: } \mu_{2,1} + \mu_{2,3} = 1 \\ &\qquad \vdots \\ &\text{Constraint 80: } -\mu_{4,5} + \mu_{5,2} - \mu_{5,4} \ge -1 \end{aligned} \tag{31}$$

Binary Variables are

$$\mu_{ij} = \{0, 1\}, \quad \text{for } 1 \le i \le 5,\ 1 \le j \le 5,\ i \ne j. \tag{32}$$

Finally, sort the results to obtain the rankings. The rankings for evaluating the five universities are given as $A_5 > A_3 > A_2 > A_4 > A_1$; i.e., EEE University is ranked the best university. CCC University, BBB University, DDD University, and AAA University are ranked the 2nd, 3rd, 4th, and 5th positions, respectively. Ranking EEE University as the best university is consistent with ARWU [66], CWTS Leiden [22], QS [67], THE [68], URAP [69], and PRSPWUN [70].

Universities operate in a highly competitive environment that is globalized. Many students travel outside their countries to study. Rankings are one of the quick references for evaluating the reputation of a university. It is good to know how students perceive these universities. These institutions have to compete with each other to attract their different audiences—students, teachers, and financiers. Students go to university with a major expectation to receive a good academic education by expert and close teachers, as well as to take advantage of the investment of time and effort of their university years. The process of higher education includes learning civic engagement, and, increasingly, it is an expectation of the students themselves. UR is an essential factor in attracting and retaining the best students, teachers, and employees [71]. The reputation of the institution is fundamental as regards teachers and staff because it generates useful bonds, produces a positive response and performance, and creates more significant commitment, involvement, and cohesion in the staff [72].

5. Conclusion

A number of people possess at least some notion of what reputation is, which they believe needs to be protected. The universities that appear in these rankings advertise their presence, while the others may question the value of the rankings. One may assume that the strain in obtaining the direct data for every university limits the number of universities ranked. There are lots of criteria to consider in evaluating a UR, and all possible criteria for the evaluation introduce more variables and uncertainties.

The core contribution of this study is a new method of evaluating alternative called the Integer Linear Programming with Grey Possibility (ILP-GP). The main idea of the ILP-GP is solving an MCDM problem as ILP problem that includes the grey possibility degrees. The ILP-GP method provides an alternative approach to solving MCDM problem as an optimization problem. More importantly, the ILP-GP eludes the difficulties of determining the grey distinguishing coefficient. Also, ILP-GP provides a fair and equal comparison of all alternatives based on all criteria. The application of the ILP-GP is not limited to the evaluation of UR. ILP-GP can be used to evaluate the business environment, real estates, financial investment, contractor selection, and so forth.

The main limitation of this research is that the evaluation of UR is based on the perception of the students. The rankings of a university based on just a sample of 1,565 students may not be enough. Students bias may lead to erroneous results, when evaluating universities around the world since every student perception is influenced by their environments. Typical academic rankings that seek to measure the quality of universities from quantitative and qualitative standpoint focus on factors such as peer reviews or publications in nature or science and address the problem of evaluating universities in different parts of the world. The hierarchical diagram is well suited for universities in the same countries. Further research can be improving the weights computation methods in the literature and applying it to the ILP-GP.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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